Section 5.2 – Counting Factors, Greatest Common Factor, Least Common Multiple
Homework (pages 206-207) problems 1-7, 8 & 9 (any method)

- Every number can be **expressed in terms of primes**
  - i.e. \( n = 2^{n_2} \cdot 3^{n_3} \cdot 5^{n_5} \cdot 7^{n_7} \cdot 11^{n_{11}} \ldots \), where \( n_2, n_3, n_5, n_{11} \) are integer exponents (maybe 0)
- **Example.** Express 144 in terms of its prime factorization
  - \( 144 = 2^4 \cdot 3^2 \). So \( n_2 = 4, n_3 = 2, \) and all the others are 0
- The **number of factors** for a given whole number is related to the exponents in its prime factorization. The number of factors = \((n_2 + 1)(n_3 + 1)(n_5 + 1)\ldots\)  
- **Example.** How many factors are there for 144? Since the factorization for 144 is \( 2^4 \cdot 3^2 \), there are \((4+1)(2+1) = 15\) different factors for 144
- You find these factors by taking all possible combinations of the prime factorization with exponent values from 0 to \( n_2 \), 0 to \( n_3 \), 0 to \( n_5 \), etc
- **Example.** What are the factors of 144? 
  - \( 2^0 \cdot 3^0 = 1 \quad 2^1 \cdot 3^0 = 2 \quad 2^2 \cdot 3^0 = 4 \quad 2^3 \cdot 3^0 = 8 \quad 2^4 \cdot 3^0 = 16 \)
  - \( 2^0 \cdot 3^1 = 3 \quad 2^1 \cdot 3^1 = 6 \quad 2^2 \cdot 3^1 = 12 \quad 2^3 \cdot 3^1 = 24 \quad 2^4 \cdot 3^1 = 48 \)
  - \( 2^0 \cdot 3^2 = 9 \quad 2^1 \cdot 3^2 = 18 \quad 2^2 \cdot 3^2 = 36 \quad 2^3 \cdot 3^2 = 72 \quad 2^4 \cdot 3^2 = 144 \)
- **Example, page 208 number 1d.** How many factors does 12\(^4\) have? 
  - Since the factorization for 12\(^4\) is \( 2^4 \cdot 3^4 \), it has \((8+1)(4+1) = 45\) factors
- **Example, page 208 number 2a.** Factor 120 into primes
  - 120 = \( 60 \cdot 2 = 30 \cdot 2^2 = 10 \cdot 3 \cdot 2^2 = 5 \cdot 3 \cdot 2^3 \)

**The Greatest Common Factor:**
- The **greatest common factor (GCF)** of two (or more) nonzero whole numbers is the largest whole number that is a factor of both (all) of the numbers
- You can find the greatest common factor by the **set intersection method**
  - Finding all factors of each of the numbers and placing them in sets
  - Finding the intersection of those sets
  - Finding the largest value in the intersection
- **Example, page 208 number 6d.** Find the GCF(42, 385)
  - Factors of 42:
    - \( 42 = 2 \cdot 21 = 2 \cdot 3 \cdot 7 \) (there are \( 2 \cdot 2 \cdot 2 = 8 \) factors)
    - \( 2^0 \cdot 3^1 \cdot 7^0 = 1 \quad 2^1 \cdot 3^1 \cdot 7^0 = 2 \quad 2^2 \cdot 3^0 \cdot 7^1 = 7 \quad 2^1 \cdot 3^0 \cdot 7^1 = 14 \)
    - \( 2^0 \cdot 3^1 \cdot 7^1 = 3 \quad 2^1 \cdot 3^1 \cdot 7^1 = 6 \quad 2^2 \cdot 3^1 \cdot 7^1 = 21 \quad 2^1 \cdot 3^1 \cdot 7^1 = 42 \)
    - \{1, 2, 3, 6, 7, 14, 21, 42\}
  - Factors of 385:
    - \( 385 = 5 \cdot 77 = 5 \cdot 7 \cdot 11 \)
    - \( 5^0 \cdot 7^0 \cdot 11^0 = 1 \quad 5^1 \cdot 7^0 \cdot 11^0 = 5 \quad 5^0 \cdot 7^0 \cdot 11^1 = 11 \quad 5^1 \cdot 7^0 \cdot 11^1 = 55 \)
    - \( 5^0 \cdot 7^1 \cdot 11^0 = 7 \quad 5^1 \cdot 7^1 \cdot 11^0 = 35 \quad 5^0 \cdot 7^1 \cdot 11^1 = 77 \quad 5^1 \cdot 7^1 \cdot 11^1 = 385 \)
    - \{1, 5, 7, 11, 35, 55, 77, 385\}
    - \{1, 2, 3, 6, 7, 14, 21, 42\} \cap \{1, 5, 7, 11, 35, 55, 77, 385\} = \{1, 7\}
    - \(\text{GCF}(42, 385) = 7\)
• You can also find the GCF by the prime factorization method
  – Find the prime factorization of each number
  – Take whatever they have in common (to the highest power possible)
• Example, page 208 number 6d. Find the GCF(42, 385)
  Factorization  $42 = 2 \cdot 21 = 2 \cdot 3 \cdot 7$
  Factorization  $385 = 5 \cdot 77 = 5 \cdot 7 \cdot 11$
  GCF(42, 385) = $7^1 = 7$
• Example, page 208 number 6f. Find the GCF(338, 507)
  Factorization  $338 = 2 \cdot 169 = 2 \cdot 13^2$
  Factorization  $507 = 3 \cdot 169 = 3 \cdot 13^2$
  GCF(338, 507) = $13^2 = 169$
• If $a$ and $b$ are whole numbers with $a \geq b$, then GCF($a, b$) = GCF($a - b, b$)
• Example, page 208 number 6f (again)
  GCF(507, 338) = GCF(507 - 338, 338) = GCF(169, 338)

Least Common Multiple:
• The least common multiple (LCM) of two (or more) nonzero whole numbers is the smallest nonzero whole number that is a multiple of each (all) of the numbers
• You can find the least common multiple with the set intersection method
  – List the nonzero multiples of each number
  – Intersect the sets
  – Take the smallest element in the intersection
• Example, page 208 number 7d. Find the LCM(66, 88)
  Multiples of 88  {88, 176, 264, 352…}
  Multiples of 66  {66, 132, 198, 264, 330…}
  LCM(66, 88) = 264
• You can also find the LCM with the prime factorization method (similar to the buildup method)
  – Express the numbers in their prime factorization
  – Take each factor (to its highest power) from each factorization, but do not repeat
• Example, page 208 number 7d (again)
  $88 = 2 \cdot 44 = 2^2 \cdot 22 = 2^3 \cdot 11$
  $66 = 3 \cdot 22 = 2 \cdot 3 \cdot 11$
  LCM(66, 88) = $2^3 \cdot 3 \cdot 11 = 264$